Diploma Programme
Programme du diplôme
Programa del Diploma

Diploma Programme
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Programa del Diploma

**Mathematics: applications and interpretation**
**Higher level**
**Paper 1**

8 May 2023

**Zone A** afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

2 hours

**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

23 pages

2223 – 7206

Please **do not** write on this page.

Answers written on this page will not be marked.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 5]

On 1 January 2022, Mina deposited $\$1000$ into a bank account with an annual interest rate of $4\%$, compounded monthly. At the end of January, and the end of every month after that, she deposits $\$100$ into the same account.

(a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places. [3]

(b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than $\$5000$ in her account. [2]

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**2.** [Maximum mark: 6]

Carys believes that, on a memory retention test, the mean score of bilingual people $(\mu_b)$ will be higher than the mean score of monolingual people $(\mu_m)$. Carys gave a memory retention test to a random sample of students in her class. The results are shown in the two tables.

| | **Scores** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Bilingual** | 100 | 94 | 100 | 90 | 100 | 94 | 98 | 98 | 98 | 98 |

| | **Scores** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Monolingual** | 97 | 92 | 88 | 98 | 88 | 94 | 100 | 100 |

Carys performs a one-tailed $t$-test at a $5\%$ level of significance. It is assumed that the scores are normally distributed and the samples have equal variances.

(a) State the null and alternative hypotheses. [2]

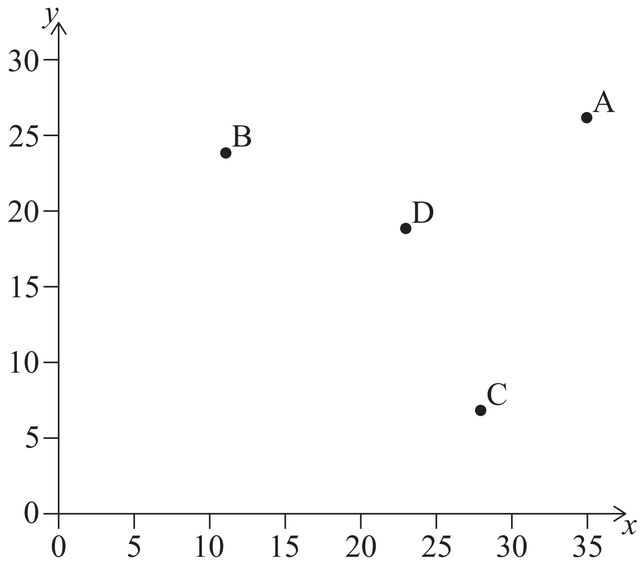
(b) Calculate the $p$-value for this test. [2]

(c) State the conclusion of the test in the context of the question. Justify your answer. [2]

**3.** [Maximum mark: 5]

Three towns have positions $A(35, 26)$, $B(11, 24)$, and $C(28, 7)$ according to the coordinate system shown where distances are measured in miles.

Dominique's farm is located at the position $D(24, 19)$.



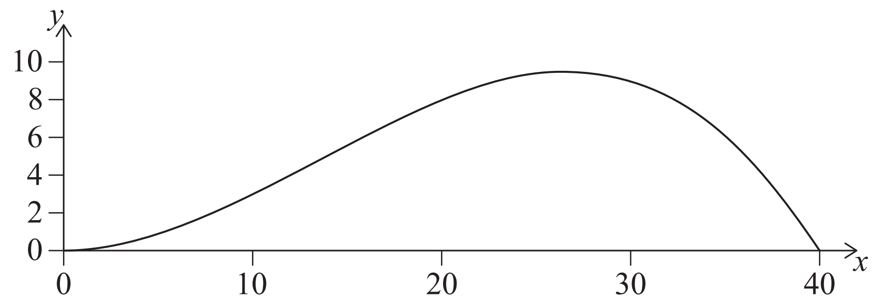

(a) Find $AD$. [2]

On a particular day, the mean temperatures recorded in each of towns $A$, $B$ and $C$ are $34\,°C$, $29\,°C$ and $30\,°C$ respectively.

(b) Use nearest neighbour interpolation to estimate the temperature at Dominique's farm on that particular day. [3]

**4.** [Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.




The heights of the model are measured at horizontal intervals and are given in the table.

| | | | | | |
|---|---|---|---|---|---|
| **Horizontal distance, $x$ cm** | 0 | 10 | 20 | 30 | 40 |
| **Vertical distance, $y$ cm** | 0 | 3 | 8 | 9 | 0 |

(a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

It is given that the equation of the curve is $y = 0.04x^2 - 0.001x^3,\ 0 \le x \le 40$.

(b) (i) Write down an integral to find the exact cross-sectional area.

(ii) Calculate the value of the cross-sectional area to two decimal places. [4]

(c) Find the percentage error in the area found using the trapezoidal rule. [2]

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**5.** [Maximum mark: 5]

A boat travels $8\text{ km}$ on a bearing of $315°$ and then a further $6\text{ km}$ on a bearing of $045°$. Find the bearing on which the boat should travel to return directly to the starting point.

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**6.** [Maximum mark: 7]

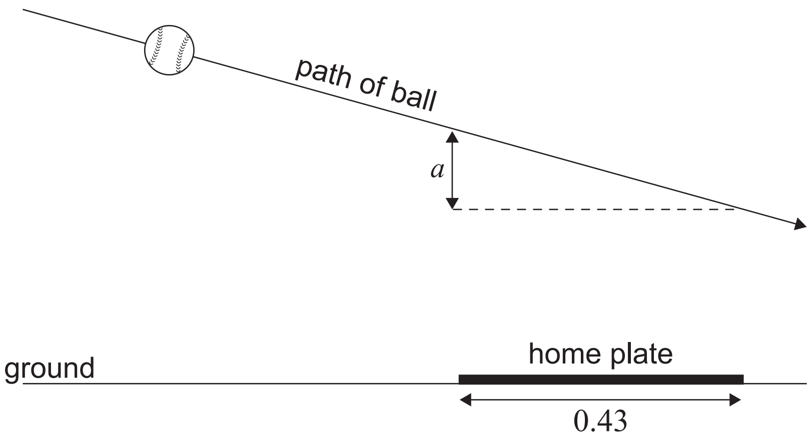
In a baseball game, Sakura is the batter standing beside home plate. The ball is thrown towards home plate along a path that can be modelled by the following function.

$$y = -0.045x + 2$$

In this model, $x$ is the horizontal distance of the ball from the point the ball is thrown and $y$ is the vertical height of the ball above the ground. Both measured in metres.

The outcome of the throw is called a strike if the height of the ball is between $0.53\text{ m}$ and $1.24\text{ m}$ at some point while it travels over home plate. The length of home plate is $0.43\text{ m}$.

**diagram not to scale**




When the ball reaches the front of home plate, the height of the ball above the ground is $1.25\text{ m}$. The height of the ball changes by $a$ metres as the ball travels over the length of home plate.

(a) (i) Find the value of $a$.

(ii) Justify why this throw is a strike. [4]

On the next throw, Sakura hits the ball towards a wall that is $5$ metres high. The horizontal distance of the wall from the point where the ball was hit is $96$ metres. The path of the ball after it is hit can be modelled by the function $h(d)$.

$$h(d) = -0.01d^2 + 1.04d + 0.66\text{, for } h,\ d > 0$$

In this model, $h$ is the height of the ball above the ground and $d$ is the horizontal distance of the ball from the point where it was hit. Both $h$ and $d$ are measured in metres.

(b) Determine whether the ball will go over the wall. Justify your answer. [3]

**(This question continues on the following page)**

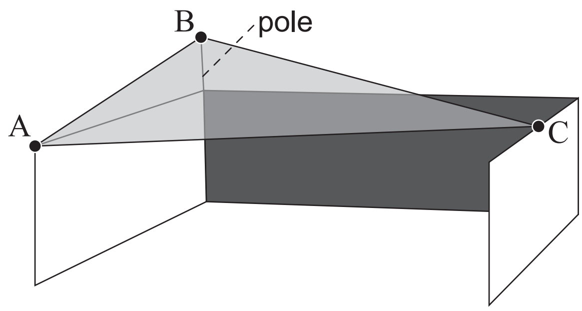
**(Question 6 continued)**

**7.** [Maximum mark: 9]

A triangular cover is positioned over a walled garden to provide shade. It is anchored at points A and C, located at the top of a $2\text{ m}$ wall, and at a point B, located at the top of a $1\text{ m}$ vertical pole fixed to a top corner of the wall.

The three edges of the cover can be represented by the vectors

$\vec{AB} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$, $\vec{AC} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$ and $\vec{BC} = \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix}$, where distances are measured in metres.




(a) Calculate the vector product $\vec{AB} \times \vec{AC}$. [2]

(b) Hence find the area of the triangular cover. [2]

The point X on [AC] is such that [BX] is perpendicular to [AC].

(c) Use your answer to part (b) to find the distance BX. [3]

(d) Find the angle the cover makes with the horizontal plane. [2]

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**8.** [Maximum mark: 7]

The random variables $(X, Y)$ follow a bivariate normal distribution with product-moment correlation coefficient $\rho$. The values of six random observations of $(X, Y)$ are shown in the table.

| $x$ | 6.3 | 4.1 | 5.6 | 9.2 | 7.8 | 8.2 |
|---|---|---|---|---|---|---|
| $y$ | 9.2 | 4.9 | 8.9 | 10.3 | 8.9 | 9.8 |

(a) State null and alternative hypotheses which could be used to test whether there is a linear correlation between $X$ and $Y$. [2]

(b) Determine the value of

(i) the product-moment correlation coefficient, $r$, of the sample.

(ii) the corresponding $p$-value. [3]

(c) State whether your result from part (b)(ii) indicates there is sufficient evidence to claim that, at the $5\%$ significance level, $X$ and $Y$ are not linearly correlated.

Give a reason for your answer. [2]

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Please **do not** write on this page.

Answers written on this page
will not be marked.

**9.** [Maximum mark: 5]

On a specific day, the speed of cars as they pass a speed camera can be modelled by a normal distribution with a mean of $67.3\,\text{km}\,\text{h}^{-1}$.

A speed of $75.7\,\text{km}\,\text{h}^{-1}$ is two standard deviations from the mean.

(a) Find the standard deviation for the speed of the cars. [2]

It is found that $82\%$ of cars on this road travel at speeds between $p\,\text{km}\,\text{h}^{-1}$ and $q\,\text{km}\,\text{h}^{-1}$, where $p < q$. This interval includes cars travelling at a speed of $74\,\text{km}\,\text{h}^{-1}$.

(b) Show that the region of the normal distribution between $p$ and $q$ is **not** symmetrical about the mean. [3]

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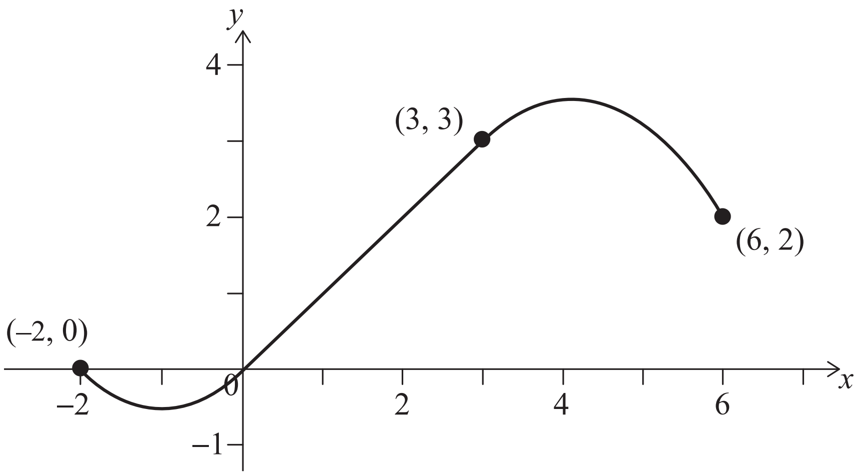
**10.** [Maximum mark: 9]

A decorative hook can be modelled by the curve with equation $y = f(x)$. The graph of $y = f(x)$ is shown and consists of a line segment from $(0, 0)$ to $(3, 3)$ and two sections formed by quadratic curves.




(a) Write down the equation of the line segment for $0 \le x \le 3$. [1]

The quadratic curve, with endpoints $(-2, 0)$ and $(0, 0)$, has the same gradient at $(0, 0)$ as the line segment.

(b) Find the equation of the curve between $(-2, 0)$ and $(0, 0)$. [3]

The second quadratic curve, with endpoints $(3, 3)$ and $(6, 2)$, has the same gradient at $(3, 3)$ as the line segment.

(c) Find the equation of this curve. [4]

(d) Write down $f$ as a piecewise function. [1]

**(This question continues on the following page)**

**(Question 10 continued)**

**11.** [Maximum mark: 6]

A shop sells oranges and lemons. The weights of the oranges are assumed to be normally distributed with mean $205$ grams and standard deviation $5$ grams. The weights of the lemons are assumed to be normally distributed with mean $105$ grams and standard deviation $3$ grams.

Nelia selects $1$ orange and $2$ lemons at random and independent of each other. Calculate the probability that the weight of her orange exceeds the combined weight of her lemons.

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**12.** [Maximum mark: 5]

Two AC (alternating current) electrical sources with the same frequencies are combined. The voltages from these sources can be expressed as $V_1 = 6\sin(at + 30°)$ and $V_2 = 6\sin(at + 90°)$.

The combined total voltage can be expressed in the form $V_1 + V_2 = V\sin(at + \theta°)$.

Determine the value of $V$ and the value of $\theta$.

**13.** [Maximum mark: 6]

The displacement, $x\,(\text{cm})$, of the end of a spring, at time $t$ (seconds), is given by

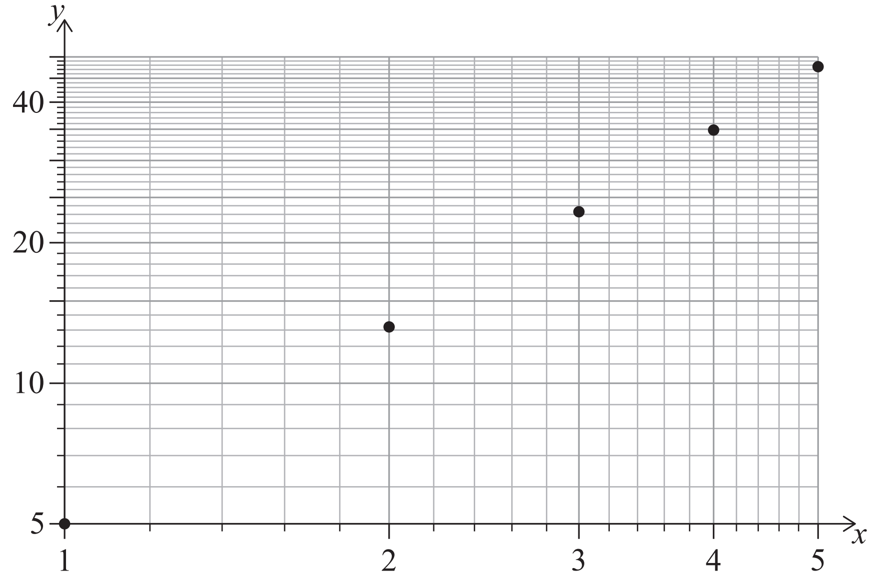
$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+2\frac{\mathrm{d}x}{\mathrm{d}t}+10x=0\,.$$

At $t=0$, $x=0.75$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=0$.

Use Euler's method, with a step length $0.1$ seconds, to estimate the value of $x$ when $t=0.5$.

**14.** [Maximum mark: 6]

Petra examines two quantities, $x$ and $y$, and plots data points on a log-log graph.

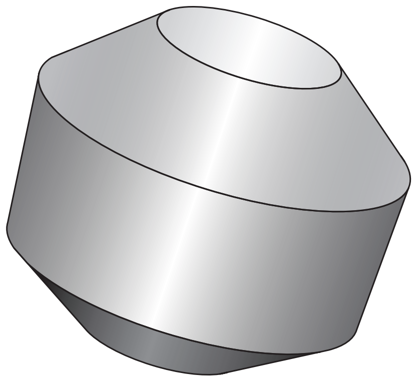



She observes that on this graph the data points follow a perfect straight line. Given that the line passes through the points $(2, 13.1951)$ and $(4, 34.822)$, find the equation of the relationship connecting $x$ and $y$. Your final answer should not include logarithms.

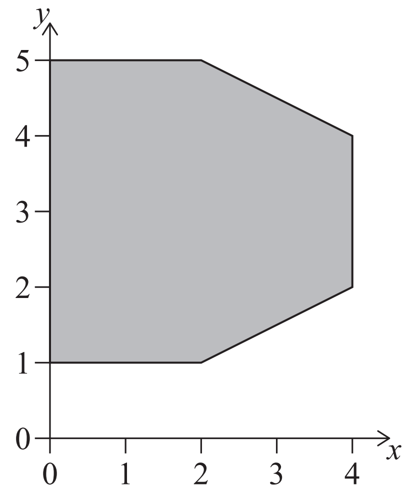
**15.** [Maximum mark: 6]

The solid shown is formed by rotating the hexagon with vertices $(2, 1)$, $(0, 1)$, $(0, 5)$, $(2, 5)$, $(4, 4)$ and $(4, 2)$ about the $y$-axis.

Solid



Hexagon




Find the volume of this solid.

**16.** [Maximum mark: 6]

The relationship between the intensity, $I$, of a light source and the distance, $d$, from the light source can be modelled by $I = \frac{k}{d^2}$.

Pablo measures the intensity of a light source at different distances. The data collected is shown in the table.

| $d$ (m) | 1 | 2 | 5 |
|---|---|---|---|
| $I$ (lm) | 42 | 11 | 1.5 |

Pablo finds the sum of square residuals in the form $1.0641k^2 - 89.62k + c$.

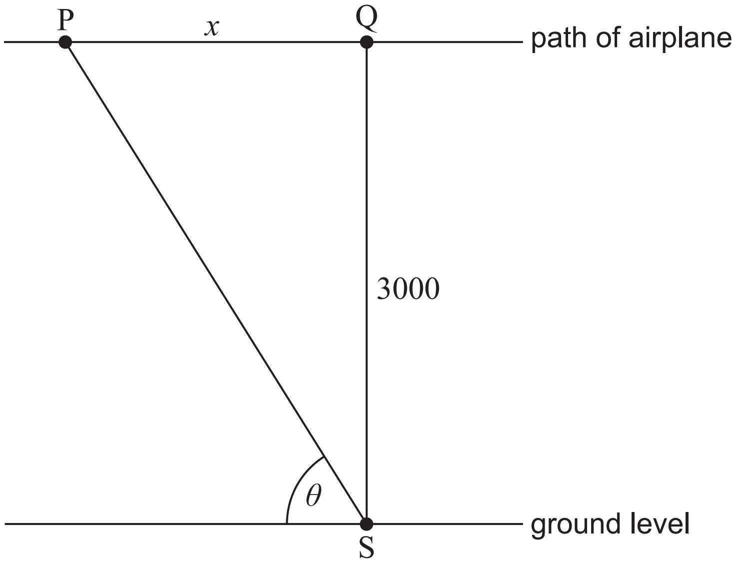
(a) Find the exact value of $c$. [4]

(b) Hence find the least squares regression curve of the form $I = \frac{k}{d^2}$. [2]

**17.** [Maximum mark: 9]

An airplane, $\mathrm{P}$, is flying at a constant altitude of $3000\,\mathrm{m}$ at a speed of $250\,\mathrm{m\,s^{-1}}$. Its path passes over a tracking station, $\mathrm{S}$, at ground level. Let $\mathrm{Q}$ be the point $3000\,\mathrm{m}$ directly above the tracking station.

At a particular time, $T$, as the airplane is flying towards $\mathrm{Q}$, the angle of elevation, $\theta$, of the airplane from $\mathrm{S}$ is increasing at a rate of $0.075$ radians per second. The distance from $\mathrm{Q}$ to $\mathrm{P}$ is given by $x$.




(a) Use related rates to show that, at time $T$, $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = -\dfrac{10\,000}{3}$. [2]

(b) Find $x(\theta)$, $x$ as a function of $\theta$. [1]

(c) Find an expression for $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ in terms of $\sin\theta$. [3]

(d) Hence find the horizontal distance from the station to the plane at time $T$. [3]

**(This question continues on the following page)**

**(Question 17 continued)**

**References:**

Please **do not** write on this page.

Answers written on this page
will not be marked.