Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Higher level

Paper 1

32 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and $1,000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a “correct” level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come “*from the use of 3 sf values*”.

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) $N = 24$

$I = 4$

$PV = \pm 1000$

$PMT = \pm 100$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

> **Note:** Award ***M1*** for an attempt to use a financial app in their technology (i.e. at least three entries seen, but not necessarily correct).
> Approaches that use the compound interest formula receive no marks.
> Award ***A1*** for correct values of PV and PMT (signs must be the same) **and** a correct value of N.

$FV = (\$)3577.43$ ***A1***

> **Note:** Award at most ***(M1)(A1)A0*** if the final answer is negative or not rounded to 2 dp.

[3 marks]

(b) $N = 36.5 \ (36.4689\ldots)$ ***(A1)***

$N = 37$ (months) ***A1***

> **Note:** Allow ***FT*** from incorrect GDC inputs seen in part (a) for the first ***A1*** providing that PV and FV have opposite signs and the resulting value of N is positive.

[2 marks]

[Total: 5 marks]

2. (a) $\mathrm{H}_0 : \mu_b = \mu_m$ ***A1***

$\mathrm{H}_1 : \mu_b > \mu_m$ ***A1***

Note: Accept equivalent statements in words such as “the **mean** score of bilingual people equals the **mean** score of monolingual people”.

[2 marks]

(b) $0.119 \ (0.119395\ldots)$ ***A2***

[2 marks]

(c) $0.119395\ldots > 0.05 \quad (11.9395\ldots\% > 5\%)$ ***R1***

(fail to reject H_0) there is insufficient evidence to suggest that bilingual people have better memory retention than monolingual people ***A1***

Note: Do not award ***R0A1***. The answer to part (c) MUST be consistent with **their** hypotheses and **their** p-value.

[2 marks]

[Total: 6 marks]

3. (a) attempt to use distance formula for points D and A ***(M1)***

$\text{DA} = \sqrt{11^2 + 7^2}$

$= 13.0$ (miles) $(13.0384\ldots,\ \sqrt{170})$ ***A1***

Note: Accept 13 miles. Award ***M0A0*** for finding the equation of the line DA. DA may be seen in part (b) but this should not be accepted as answer for part (a).

[2 marks]

(b) $\left(\text{DB} = \sqrt{13^2 + 5^2} =\right)\ 13.9\ \left(13.9283...,\ \sqrt{194}\right)$ **AND**

$\left(\text{DC} = \sqrt{4^2 + 12^2} =\right)\ 12.6\ \left(12.6491...,\ \sqrt{160}\right)$ ***A1***

recognizing closest town is best estimate ***(M1)***

(town C is closest)

$30\ ^\circ\text{C}$ ***A1***

Note: If their DA from part (a) is the shortest length, then allow ***FT*** in (b).

[3 marks]

[Total: 5 marks]

4. (a) attempt to substitute $h = 10$ and at least two different values of y into the trapezoidal rule ***(M1)***

$$\frac{10}{2}\left((0+0)+2(3+8+9)\right)$$

$$= 200\ (\text{cm}^2)$$ ***A1***

[2 marks]

(b) (i) $\int_0^{40} 0.04x^2 - 0.001x^3 \mathrm{d}x$ **OR** $\int_0^{40} y\,\mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral (including $\mathrm{d}x$), ***A1*** for correct limits in the correct location.

(ii) $213.33\ (\text{cm}^2)$ ***A2***

Note: Answer must be given to 2 decimal places to award ***A2***. Award ***A1A0*** for a correct answer given to an incorrect accuracy of at least 3 significant figures, e.g. $213\ (\text{cm}^2)$.

[4 marks]

(c) attempt to substitute their parts (a) and (b)(ii) into percentage error formula ***(M1)***

$$\left|\frac{213.333... - 200}{213.333...}\right| \times 100$$

$$= 6.25(\%)\ \ (6.24999...(\%))$$ ***A1***

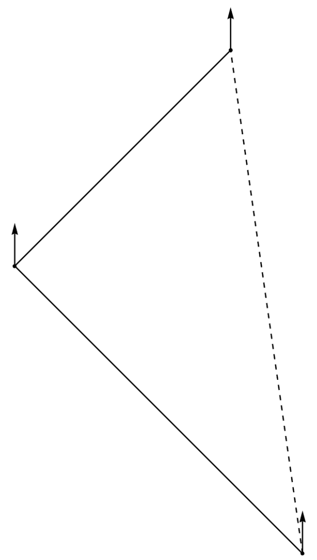
Note: Award ***(M1)A0*** for a final answer of $-6.25(\%)$ or 0.0625.

[2 marks]

[Total: 8 marks]

5. **METHOD 1**

diagram showing (approximately) correct directions (and order) for the 315° and 045° ***(A1)***



recognizing right angle triangle ***(M1)***

correct expression to find second angle in triangle ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

correct expression to find bearing ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)+135^\circ$ **OR** $360^\circ-\left(\arctan\left(\frac{8}{6}\right)+135^\circ\right)$

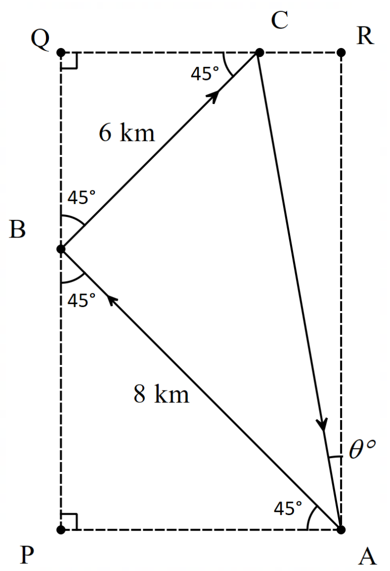
$=172^\circ \quad (171.869...^\circ)$ ***A1***

continued…

Question 5 continued

METHOD 2

diagram showing (approximately) correct directions (and order) for the $315°$ and $045°$ (these may be shown in reverse as the return journey) ***(A1)***



finding the lengths marked AP, BP, CQ and BQ in the diagram ***(M1)***

$$\text{AP} = \text{BP} = 8\frac{\sqrt{2}}{2} = 5.6568...$$

$$\text{CQ} = \text{BQ} = 6\frac{\sqrt{2}}{2} = 4.2426...$$

Note: This may be done using a vector approach.

using $\tan\theta° = \dfrac{\text{AP} - \text{CQ}}{\text{PB} + \text{BQ}}$ or equivalent to find the direction of AC ***(A1)***

correct expression to find bearing ***(A1)***

$$180° - \arctan\left(\frac{8\frac{\sqrt{2}}{2} + 6\frac{\sqrt{2}}{2}}{8\frac{\sqrt{2}}{2} - 6\frac{\sqrt{2}}{2}}\right)$$

$= 172° \quad (171.869...°)$ ***A1***

[Total: 5 marks]

6. (a) (i) **METHOD 1**

attempt to find change in height of the ball using gradient ***(M1)***

$$\frac{a}{0.43} = (-)\,0.045$$

$$a = (-)\,0.045 \times 0.43$$

$$a = (-)\,0.0194\,(\text{m}) \quad (0.01935\ (\text{m}))$$ ***A1***

METHOD 2

attempt to find height at back of home plate ***(M1)***

horizontal distance to the front of the home plate $= 16.6666\ldots\ (\text{m})$

height at the back of the home plate $= -0.045(16.6666\ldots + 0.43) + 2$

$(= 1.23065\ (\text{m}))$

Note: The ***M1*** can be awarded for $16.6666\ldots + 0.43$ seen at some point.

$(a = 1.25 - 1.23065\ldots)$

$(a =)\ (-)0.0194\ (\text{m}) \quad (0.01935\ (\text{m}))$ ***A1***

(ii) $1.25 - 0.01935 = 1.23065$ (may be seen in part (a)(i)) ***A1***

$0.53 < 1.23065 < 1.24$ ***R1***

therefore a strike ***AG***

Note: Do not award ***A0R1***.

[4 marks]

continued…

Question 6 continued

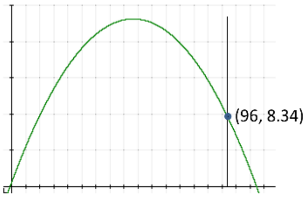
(b) **METHOD 1**

indication of $d = 96$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$\left(h(96)=\right) -0.01(96)^2 + 1.04(96) + 0.66$

OR



THEN

$\left(h(96)=\right)\ 8.34\ (\text{m})$ ***A1***

$8.34 > 5$ so the ball will go over the wall. ***A1***

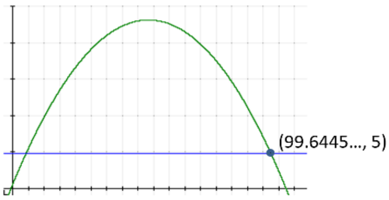
METHOD 2

indication of $h = 5$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$5 = -0.01d^2 + 1.04d + 0.66$

OR



THEN

$d = 99.6\ (\text{m})\ \left(99.6445\ldots\ (\text{m})\right)\ \left(d = 4.35548\ldots\ (\text{m})\right.$ may also be seen$\left.\right)$ ***A1***

$96 < 99.6445\ldots$ so the ball will go over the wall. ***A1***

[3 marks]

[Total: 7 marks]

7. (a) attempt to find the vector product (e.g. one term correct) ***(M1)***

$$\begin{pmatrix}0\\6\\1\end{pmatrix}\times\begin{pmatrix}7\\3\\0\end{pmatrix}=\begin{pmatrix}-3\\7\\-42\end{pmatrix}$$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to use the vector product formula for the area of triangle

(condone incorrect signs and missing $\frac{1}{2}$) ***(M1)***

area $=\frac{1}{2}\sqrt{3^2+7^2+42^2}$

$=21.3\ (\text{m}^2)\ \left(21.3424\ldots,\ \frac{1}{2}\sqrt{1822}\right)$ ***A1***

METHOD 2

find θ using $\vec{AB}\times\vec{AC}=\left|\vec{AB}\right|\left|\vec{AC}\right|\sin\theta$ ***(M1)***

$\theta=67.1^\circ\ \ (67.1350^\circ\ldots,\ 1.171728\ldots$ radians)

then area $=\frac{1}{2}\left|\vec{AB}\right|\left|\vec{AC}\right|\sin\theta$

$=21.3\ (\text{m}^2)\ \left(21.3424\ldots,\ \frac{1}{2}\sqrt{1822}\right)$ ***A1***

[2 marks]

continued…

Question 7 continued

(c) $\text{AC} = 7.61577... \ \left(\sqrt{58}\right)$ ***(A1)***

setting the area formula $\frac{1}{2} \times \text{base} \times \text{height}$ equal to their part (b) ***(M1)***

$$\text{BX} = \frac{2 \times 21.3424...}{\sqrt{58}}$$

$= 5.60 \ \ (5.60480...)$ ***A1***

Note: Award ***A1*** for 5.6.
Award ***A1*** for 5.59 (5.5936…) from the use of 21.3 to 3 sf.

[3 marks]

(d) attempting to set up a trig ratio ***(M1)***

angle is $\arcsin\left(\frac{1}{\text{BX}}\right)$

$10.3^\circ \ \ (10.2776...^\circ, \ 0.179378\ldots$ radians) ***A1***

[2 marks]

[Total: 9 marks]

8. (a) H_0: X and Y are not (linearly) correlated **OR** $\rho = 0$ ***A1***

H_1: X and Y are (linearly) correlated **OR** $\rho \neq 0$ ***A1***

Note: Accept "independent" or "not associated" in place of "not correlated".
If H_0 and H_1 are reversed, then award ***A0A1***.

[2 marks]

(b) (i) $r = 0.849$ $(0.848886...)$ ***A1***

(ii) p-value $= 0.0325$ $(0.0325277...)$ ***A2***

Note: Award ***A1*** for p-value $= 0.033$ or p-value $= 0.03$.
Award ***FT*** for $\rho > 0$ or $\rho < 0$ in part (a), p-value $= 0.0163$ $(0.0162638\ldots)$
or p-value $= 0.984$ $(0.983736\ldots)$
Award the full marks for seeing the values of r and p-value from the markscheme when H_0 and H_1 are reversed in part (a).

[3 marks]

(c) $0.0325 < 0.05$ ***R1***

(so we reject H_0 in favour of H_1)

(there is sufficient evidence to suggest) X and Y are (linearly) correlated ***A1***

Note: Their conclusion must be consistent with their p-value and their hypotheses and it must be in context.

[2 marks]

[Total: 7 marks]

9. (a) attempt to find the difference between 75.7 and 67.3 ***(M1)***

$\frac{75.7-67.3}{2}$

$4.2\ (\text{km h}^{-1})$ ***A1***

[2 marks]

(b) **METHOD 1 (Comparing areas above and below the mean)**

$P(67.3 <$ speed $< 74)$ **OR** Normal CDF(67.3, 74, 67.3, 4.2) **OR** sketch of normal distribution with 67.3 and 74 labelled and shaded between ***(M1)***

area of region between mean and q is at least $0.445\ (0.444670\ldots)$ ***A1***

Hence no more than $0.375\ (0.375329\ldots)$ between mean and p ***R1***

The region between p and q is not symmetrical ***AG***

METHOD 2 (Comparing areas in the tails)

attempt to calculate probability that speed $< p$ and speed $> q$ with $q = 74$ ***(M1)***

$P(\text{speed} < 74) = 0.944670\ldots$

$P(\text{speed} < p) = (0.944670\ldots - 0.82 =)\ 0.124670\ldots$

$P(\text{speed} > q) = (1 - 0.944670\ldots =)\ 0.0553295\ldots$ ***A1***

if $q \geq 74$, then $P(\text{speed} > q) \leq 0.0553295$ and $P(\text{speed} < p) \geq 0.124670$ so

$P(\text{speed} > q)$ will never equal $P(\text{speed} < p)$ ***R1***

the region between p and q is not symmetrical ***AG***

continued…

Question 9 continued

METHOD 3 (Assumption of symmetry comparing speeds)

attempt to calculate area below q assuming distribution is symmetrical ***(M1)***

e.g. $\mathrm{P}(\text{speed} < q) = 0.82 + ½ \times 0.18 \quad (0.91)$

EITHER

$(q =)\ 72.9 \quad (72.9311\ldots)$ ***A1***

$72.9 < 74$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$\mathrm{P}(\text{speed} < 74) = 0.945 \quad (0.944670\ldots)$ ***A1***

$0.945 > 0.91$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

METHOD 4 (Assumption of symmetry comparing areas)

attempt to calculate symmetrical area with 74 as a boundary ***(M1)***

$\mathrm{P}(60.6 < \text{speed} < 74)$ **OR** Normal CDF$(60.6, 74, 67.3, 4.2)$ **OR**

$\mathrm{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF$(67.3, 74, 67.3, 4.2)$

EITHER

$0.889 \ (0.889340\ldots)$ ***A1***

$0.889 > 0.82$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$0.445 \ (0.444670\ldots)$ ***A1***

$0.445 > 0.82 \div 2$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

[3 marks]

[Total: 5 marks]

10. (a) $y = x$ ***A1***

[1 mark]

(b) **METHOD 1**

equation has the form $y = ax^2 + bx + c$

when $x = 0$, $y = 0$ so $c = 0$

$\frac{dy}{dx} = 2ax + b$

attempt to find the value of b by setting *their* derivative equal to 1 when x is 0 ***(M1)***

$2a(0) + b = 1$

$b = 1$ ***(A1)***

when $x = -2$, $y = 0$

$a = \frac{1}{2}$ (and hence $y = \frac{1}{2}x^2 + x$) ***A1***

METHOD 2

equation has the form $y = ax(x+2)$ **OR** $y = ax^2 + 2ax$ ***A1***

$\frac{dy}{dx} = 2ax + 2a$

attempt to find the value of a by setting *their* derivative equal to 1 when x is 0 ***(M1)***

$a = \frac{1}{2}$ (and hence $y = \frac{1}{2}x^2 + x$) ***A1***

Note: Writing $y = x(x+2)$ is incorrect and gains no marks.

[3 marks]

continued…

Question 10 continued

(c) equation is $y = ax^2 + bx + c$

finding an expression for $\frac{dy}{dx}$ with unknown coefficients ***(M1)***

$$\frac{dy}{dx} = 2ax + b$$

setting up two equations using two points AND/OR one equation using the gradient function ***(M1)***

three correct equations ***(A1)***

$$9a + 3b + c = 3$$
$$36a + 6b + c = 2$$
$$6a + b = 1$$

$a = -\frac{4}{9},\ b = \frac{11}{3},\ c = -4$ ($a = -0.444444...,\ b = 3.66666...,\ c = -4$) ***A1***

(and hence $y = -\frac{4}{9}x^2 + \frac{11}{3}x - 4$)

[4 marks]

(d) $f(x) = \begin{cases} \frac{1}{2}x^2 + x & , \quad -2 \le x < 0 \\ x & , \quad 0 \le x \le 3 \\ -\frac{4}{9}x^2 + \frac{11}{3}x - 4 & , \quad 3 < x \le 6 \end{cases}$ ***A1***

Note: Condone open or closed endpoints for all intervals.
Condone y in place of $f(x)$.
Allow ***FT*** from parts (a), (b) and (c).

[1 mark]

[Total: 9 marks]

11. Let $D = O - L - L$ ***(A1)***

(mean =) $205 - 105 - 105 \ (= -5)$ ***(A1)***

manipulating variances (not standard deviations) ***(M1)***

(variance =) $25 + 9 + 9 \ (= 43)$ **OR** (SD =) $6.55743...$ ***(A1)***

$D \sim \mathrm{N}(205 - 105 - 105,\ 25 + 9 + 9)$

attempt to find the probability that $D > 0$ ***(M1)***

$\mathrm{P}(D > 0)$

$= 0.223 \ (0.222882...)$ ***A1***

Note: If $D = O - 2L$ is seen or implied, award at most ***(A0)A1(M0)(A0)M1A0***.

[Total: 6 marks]

12. METHOD 1 Analytical approach

attempt to express V_1 or V_2 in exponential form ***(M1)***

e.g. $V_1 = \mathrm{Im}(6\,\mathrm{e}^{\mathrm{i}\left(at+\frac{\pi}{6}\right)}),\ V_2 = \mathrm{Im}(6\,\mathrm{e}^{\mathrm{i}\left(at+\frac{\pi}{2}\right)})$

Note: Accept angles in radians or degrees.

$(V_1 + V_2 =)\ 6\mathrm{e}^{\mathrm{i}\times\frac{\pi}{6}} + 6\mathrm{e}^{\mathrm{i}\times\frac{\pi}{2}}$ ***(A1)***

Note: This mark can be awarded even if seen as part of a correct larger expression.

$= 10.4\mathrm{e}^{1.05\mathrm{i}}\ \left(6\sqrt{3}\,\mathrm{e}^{\frac{\mathrm{i}\pi}{3}}\right)$ ***(A1)***

so V is 10.4 $(10.3923...,\ 6\sqrt{3})$ and θ is 60 (degrees) ***A1A1***

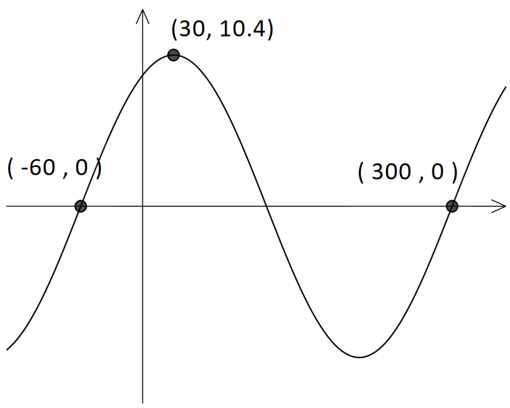
Note: Accept any value for θ that rounds to a 2sf answer of 60. Do **not** accept a final answer for an angle in radians. Do **not** award ***A1*** for answer of 60° resulting from incorrect working.

continued…

Question 12 continued

METHOD 2 Graphical approach

let $at = x$ and plot $V_1 + V_2$ curves on GDC ***(M1)***



attempt to find maximum ***(M1)***

$V = 10.4$ ***A1***

attempt to find any x-axis intercept (either -60 or 300) ***(M1)***

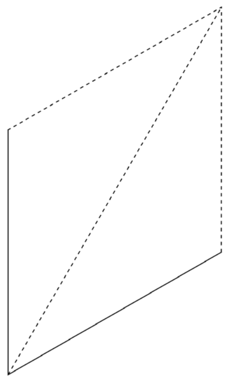
$\theta = 60$ (degrees) $\left(\theta = -300 \text{ (degrees)}\right)$ ***A1***

continued…

Question 12 continued

METHOD 3 Geometric approach

considering the rhombus



(M1)

$V = \sqrt{6^2 + 6^2 - 2 \times 6 \times 6 \cos 120^\circ}$ ***(M1)***

$\left(= \sqrt{108} = 6\sqrt{3}\right) = 10.4 \ \ (10.3923\ldots)$ ***A1***

$\theta = 60$ (degrees) ***A2***

Note: An answer of $\theta = -300$. is most likely to be seen in METHOD 2, but should be condoned in METHODS 1 and 3 if seen there.

[Total: 5 marks]

13. $\frac{dx}{dt} = y$ ***(A1)***

$\frac{dy}{dt} = -10x - 2y$ ***(A1)***

> **Note:** Writing $\frac{d^2x}{dt^2} = -10x - 2\frac{dx}{dt}$ is a valid approach and should be awarded ***A1A1***.

attempt to use the Euler equations shown by finding either a correct x_{n+1} or y_{n+1} ***(M1)***

correct equations for both x_{n+1} and y_{n+1} ***(A1)***

$x_{n+1} = x_n + 0.1(y_n)$, $y_{n+1} = y_n + 0.1(-10x_n - 2y_n)$ (accept equivalent notation)

$(t_{n+1} = t_n + 0.1)$

> **Note:** All of the above marks can be implied by a correct second row in a table **OR** by a correct f_1 and f_2 clearly identified for use in Euler's method formula.

T	x	y
0	0.75	0
0.1	**0.75**	**-0.75**
0.2	0.675	-1.35
0.3	0.54	-1.755
0.4	0.3645	-1.944
0.5	0.1701	

so estimate is 0.170 ***A2***

> **Note:** Accept 0.17 rounded to 2 sf.

[Total: 6 marks]

14. METHOD 1 Analytical approach

recognizing that the linear equation must be expressed in log form ***(M1)***

$\log y = m\log x + \log c$ (or $\log y = m\log x + C$)

EITHER

use of slope formula (must involve logs) ***(M1)***

$m = \dfrac{\log(34.822) - \log(13.1951)}{\log(4) - \log(2)} = 1.4$ ***A1***

attempt to substitute a value ***(M1)***

$\log c = \log(13.1951) - 1.4\log 2 (= 0.69897...)$

$\Rightarrow c = 5$ ***A1***

OR

$y = c.x^m$ ***(A1)***

attempt to set up two equations involving power functions ***(M1)***

$13.1951 = c \times 2^m$ and $34.822 = c \times 4^m$

$2^m = \dfrac{34.822}{13.1951} = 2.639... \Rightarrow m = \log_2 2.639... = 1.4$ ***A1***

$c = \dfrac{13.1951}{2.639...} = 5$ ***A1***

THEN

(so the equation is) $y = 5 \times x^{1.4}$ ***A1***

METHOD 2 Regression analysis

recognizing that a log-log graph results in a power function model ***(M1)***

$y = a \times x^b$

attempt to find a power regression model using the given two points ***(M1)***

$a = 5$ and $b = 1.4$ ***(A1)(A1)***

(so the equation is) $y = 5 \times x^{1.4}$ ***A2***

[Total: 6 marks]

15. METHOD 1 Using the volume formula

volume of a "full" or "half" cylinder (seen anywhere) ***(A1)***

$\pi\int_2^4 4^2 \,\mathrm{d}y$, $\pi\times 4^2\times 2$, 32π $(100.53\ldots)$ **OR**

$\pi\int_2^3 4^2 \,\mathrm{d}y$, $\pi\times 4^2\times 1$, 16π $(50.265\ldots)$

one correct equation for the diagonal lines (seen anywhere) ***(A1)***

$y=\frac{1}{2}x$ or $y=6-\frac{1}{2}x$

attempt to write one equation x in terms of y ***(M1)***

$x=2y$, $x=12-2y$

EITHER (symmetry plus the volume of the "half" cylinder)

recognition of symmetry between $y=1$ and $y=3$ ***(M1)***

$2\pi\left(\int_1^2 (2y)^2 \,\mathrm{d}y+\int_2^3 4^2 \,\mathrm{d}y\right)$ ***(A1)***

OR (symmetry plus volume of the "full" cylinder)

recognition of symmetry between $y=1$ and $y=2$ ***(M1)***

$2\pi\left(\int_1^2 (2y)^2 \mathrm{d}y\right)+\int_2^4 4^2 \mathrm{d}y$ ***(A1)***

OR (calculation of separate parts) ***(M1)***

$\pi\left(\int_1^2 (2y)^2 \,\mathrm{d}y+\int_2^4 4^2 \,\mathrm{d}y+\int_4^5 (-2y+12)^2 \,\mathrm{d}y\right)$ ***(A1)***

THEN

(volume of the solid=) 159 $\left(159.174..., \frac{152\pi}{3}\right)$ ***A1***

continued…

Question 15 continued

METHOD 2 Geometric approach using cones and cylinders

volume of a cylinder (seen anywhere) ***(A1)***

$\pi\times 4^2\times 2,\ 32\pi\ \ (100.53\ldots)$ (a *full* cylinder) **OR**

$\pi\times 4^2\times 1,\ 16\pi\ \ (50.265\ldots)$ (a *half* cylinder)

using volume of cone formula to find the volume of the truncated cone ***(M1)***

correct expression to find the volume of the truncated cone (seen anywhere) ***(A1)***

$$\frac{1}{3}\left(\pi\times 4^2\times 2-\pi\times 2^2\times 1\right)$$

attempt to find an expression for total volume using symmetry or individual parts ***(M1)***

correct expression for total volume ***(A1)***

$$2\left(\frac{1}{3}\left(\pi 4^2\times 2-\pi 2^2\times 1\right)+\pi 4^2\times 1\right)\quad \textbf{OR}\quad \frac{1}{3}\left(\pi 4^2\times 2-\pi 2^2\times 1\right)+\pi 4^2\times 2+\frac{1}{3}\left(\pi 4^2\times 2-\pi 2^2\times 1\right)$$

(volume of the solid=) $159\ \left(159.174...,\ \dfrac{152\pi}{3}\right)$ ***A1***

Note: There are other valid approaches possible.

[Total: 6 marks]

16. (a) attempt to find what the model predicts in terms of k ***(M1)***

$k, \frac{k}{4}, \frac{k}{25}$

correct expression for sum of square residuals ***(A1)***

$(k-42)^2 + \left(\frac{k}{4} - 11\right)^2 + \left(\frac{k}{25} - 1.5\right)^2$

valid attempt to find c by expanding or recognizing the constant terms ***(M1)***

$c = 42^2 + 11^2 + 1.5^2$

$= 1887.25$ ***A1***

[4 marks]

(b) valid method to find the k value at the minimum ***(M1)***

$k = \frac{89.62}{2 \times 1.0641}$ $(= 42.1107...)$, graph, completing the square

(so least squares regression is) $I = \frac{42.1}{d^2}$ ***A1***

[2 marks]

[Total: 6 marks]

17. (a) attempt to use the chain rule to set up a related rate ***(M1)***

correct expression ***A1***

$\frac{dx}{d\theta} = \frac{dx}{dt} \div \frac{d\theta}{dt}$ **OR** $\frac{-250}{0.075}$

$= -\frac{10000}{3}$ ***AG***

[2 marks]

(b) $x(\theta) = \frac{3000}{\tan\theta}$ ***A1***

[1 mark]

(c) attempt to use chain rule **OR** quotient rule ***(M1)***

$\frac{-3000}{\tan^2\theta \times \cos^2\theta}$, $\frac{-3000\left(\sin\theta(-\sin\theta) - \cos^2\theta\right)}{\sin^2\theta}$ ***(A1)***

$= -\frac{3000}{\sin^2\theta}$ ***A1***

[3 marks]

(d) setting their equation in part (c) equal to the given expression in part (a) ***(M1)***

$-\frac{3000}{\sin^2\theta} = -\frac{10000}{3}$

$\theta = 1.24904...$ ***(A1)***

$x(1.24904...) = 1000\,\text{m}$ ***A1***

[3 marks]

[Total: 9 marks]