Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Standard level

Paper 1

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a)

	Event		Rank	
Country	**Long Jump (m)**	**High Jump (m)**	**Long Jump Rank**	**High Jump Rank**
Germany	7.64	2.11	1	**1**
France	7.52	2.08	2	**2**
Estonia	7.49	1.84	3	**10**
Canada	7.44	2.02	4	**4.5**
Netherlands	7.33	2.05	5	**3**
Ukraine	7.28	2.02	6	**4.5**
Algeria	7.22	1.90	7	**8**
Austria	7.11	1.87	8	**9**
Grenada	6.98	1.99	9	**6**
Japan	6.64	1.96	10	**7**

A1A1

Note: Award ***A1*** for ranking of tied heights, ***A1*** for correct ranking of non-tied heights.

[2 marks]

(b) $(r_s =) \ 0.541 \quad (0.541035\ldots)$ ***A2***

Note: Award ***A2*** for an answer of 0.539 $(0.539393\ldots)$ from use of the formula for Spearman's rank correlation coefficient when data has tied ranks.

[2 marks]

(c) moderate (correlation) ***A1***

as long jump ranking increases, high jump ranking will (likely) increase ***A1***

[2 marks]

[Total: 6 marks]

2. (a) attempt to calculate $\hat{AHB}$ using 33 **OR** use of alternate angles ***(M1)***

e.g., $180-(33+130)$ **OR** $90-(33+40)$ **OR** $57-40$

$17\ (°)$ ***A1***

[2 marks]

(b) attempt to use sine rule ***(M1)***

$$\frac{BH}{\sin(130^\circ)}=\frac{156}{\sin(17^\circ)}$$ ***(A1)***

$(BH=)\ 409\ (m)\ (408.736\ldots)$ ***A1***

Note: If radians are used, answer is $151\ (150.922\ldots)$; award at most ***(M1)(A1)A0***.

[3 marks]

(c) (the angle of depression from the hot air balloon) gets smaller ***A1***
(as the horizontal distance increases)

[1 mark]

[Total: 6 marks]

3. (a) $N = 24$

$I = 4$

$PV = \pm 1000$

$PMT = \pm 100$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (i.e. at least three entries seen, but not necessarily correct).
Approaches that use the compound interest formula receive no marks.
Award ***A1*** for correct values of PV and PMT (signs must be the same) **and** a correct value of N.

$FV = (\$)3577.43$ ***A1***

Note: Award at most ***(M1)(A1)A0*** if the final answer is negative or not rounded to 2 dp.

[3 marks]

(b) $N = 36.5 \ (36.4689\ldots)$ ***(A1)***

$N = 37$ (months) ***A1***

Note: Allow ***FT*** from incorrect GDC inputs seen in part (a) for the first ***A1*** providing that PV and FV have opposite signs and the resulting value of N is positive.

[2 marks]

[Total: 5 marks]

4. (a) $H_0 : \mu_b = \mu_m$ ***A1***

$H_1 : \mu_b > \mu_m$ ***A1***

Note: Accept equivalent statements in words such as "the **mean** score of bilingual people equals the **mean** score of monolingual people".

[2 marks]

(b) $0.119 \ (0.119395\ldots)$ ***A2***

[2 marks]

(c) $0.119395\ldots > 0.05 \ \ (11.9395\ldots\% > 5\%)$ ***R1***

(fail to reject H_0) there is insufficient evidence to suggest that bilingual people have better memory retention than monolingual people ***A1***

Note: Do not award ***R0A1***.
The answer to part (c) MUST be consistent with **their** hypotheses and **their** p-value.

[2 marks]

[Total: 6 marks]

5. (a) 2 ***A1***

[1 mark]

(b) attempt to substitute their part (a) and point $(3, -1)$ into the slope-intercept form or point-slope form of an equation ***(M1)***

$-1 = 2 \times 3 + c$ **OR** $y + 1 = 2(x - 3)$

$y = 2x - 7$ ***A1***

Note: Equation must be in the form $y = mx + c$ for ***A1*** to be awarded.

[2 marks]

continued…

Question 5 continued

(c) **METHOD 1**

attempt to show that P does not lie on L_2 ***(M1)***

e.g. $-\frac{1}{2}(3)-\frac{5}{2}$ **OR** graph showing L_2 and P in approximate correct locations

$-1 \neq -\frac{1}{2}(3)-\frac{5}{2}$ $(-1 \neq -4)$ **OR** $(3, -1)$ does not lie on the graph of L_2 ***R1***

hence L_2 is not the normal line to $f(x)$ at point P ***AG***

METHOD 2

attempt to find the equation of the normal line at $(3, -1)$ ***(M1)***

$(-1 = -\frac{1}{2}(3)+c$ **OR** $y+1 = -\frac{1}{2}(x-3))$

the normal line is $y = -\frac{1}{2}x+\frac{1}{2}$ ***R1***

hence L_2 is not the normal line to $f(x)$ at point P ***AG***

METHOD 3

attempt to find the intersection of L_1 and L_2 ***(M1)***

Intersection of $y = 2x-7$ and $y = -\frac{1}{2}x-\frac{5}{2}$ is $(1.8, -3.4)$

$x = 1.8 \neq 3$ **OR** $y = -3.4 \neq -1$ ***R1***

hence L_2 is not the normal line to $f(x)$ at point P ***AG***

Note: Accept equivalent written arguments provided values are seen.
Methods 1 and 2 are independent of the answers in (a) and (b) but FT marks can be given for Method 3.

[2 marks]

[Total: 5 marks]

6. (a) attempt to set up a direct variation equation that includes a constant, k, or the calculation of a constant using 12.3 and 50 ***(M1)***

e.g., $d = kv^2$ **OR** $12.3 = k \times 50^2$

$(k =)\ 0.00492 \quad \left(\frac{1}{203.252...}\right)$

$d = 0.00492v^2$ **OR** $d = \frac{v^2}{203}$ ***A1***

[2 marks]

(b) substituting 33 for d in their part (a) ***(A1)***

$33 = 0.00492 \times v^2$ **OR** $33 = \frac{v^2}{203.252...}$

$(v =)\ 81.9\ (\text{km h}^{-1}) \quad (81.8982\ldots\ (\text{km h}^{-1}))$ ***A1***

[2 marks]

(c) *Award **R1** for a reasonable variable that exists after the brakes are applied such as:*

- road material
- weather conditions
- condition/type of brakes
- weight/type of vehicle
- gradient/incline of road
- traction
- wind resistance
- friction

R1

Note: Do not accept a variable that refers to the timing of the brakes being applied such as:

- slow reaction time
- inexperienced driver

[1 mark]

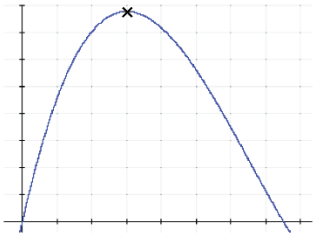
[Total: 5 marks]

7. (a) $(k =)\ 15$ ***A1***

[1 mark]

(b) **EITHER**

attempt to sketch the function $V(x)$ with indication of maximum ***(M1)***



OR

recognition of setting the derivative to 0 ***(M1)***

e.g. $V'(x) = 0$

THEN

$(x =)\ 6\ (\text{cm})$ ***A1***

Note: Award ***(M1)A0*** for the maximum given as a coordinate pair.

[2 marks]

(c) 44 or 26 seen ***(A1)***

attempting to adjust the constant(s) in the given volume formula ***(M1)***

volume of second box $= (44 - 2x)(26 - 2x)(x)$

(New maximum volume $=$) $2730\ \text{cm}^3$ $(2726.13...\ \text{cm}^3)$ ***A1***

Note: Units must be seen to award the final ***A1***. Award ***(A1)(M1)A0*** for the maximum given as a coordinate pair.

[3 marks]

[Total: 6 marks]

8. (a) attempt to substitute 5000 for G ***(M1)***

$0.301p = \log_{10} 5000$

$(p =)\ 12.3$ (bits) $(12.2889\ldots)$ ***A1***

[2 marks]

(b) $(G =) 10^{0.301p}$ **OR** 2^p ***A1***

[1 mark]

(c) attempt to substitute 28 for p in given equation or $G(p)$ ***(M1)***

$0.301 \times 28 = \log_{10} G$ **OR** $(G =)\ 10^{0.301 \times 28}$

$(G =)\ 2.68 \times 10^8$ $(2.67916\ldots \times 10^8)$ ***A1A1***

Note: Award ***A1*** for 2.68, ***A1*** for 10^8. Award ***M1A1A0*** for a correct final answer not written in scientific notation or written incorrectly in scientific notation (e.g., $268\ 000\ 000$ or 26.8×10^7 or 2.68E08).

[3 marks]

(d) if a password has an **entropy of 0** (bits), then the password can be **guessed in one** try / then the **password is known** ***R1***

Note: Reference must be made to both entropy and number of guesses/password known for ***R1*** to be awarded.
Do not accept "no password" as this contradicts the context.

[1 mark]

Total [7 marks]

9. (a) attempt to substitute $h = 10$ and at least two different values of y into the trapezoidal rule ***(M1)***

$$\frac{10}{2}\big((0+0)+2(3+8+9)\big)$$

$$= 200\ (\text{cm}^2)$$ ***A1***

[2 marks]

(b) (i) $\int_0^{40} 0.04x^2 - 0.001x^3 \,\text{d}x$ **OR** $\int_0^{40} y \,\text{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral (including $\text{d}x$), ***A1*** for correct limits in the correct location.

(ii) $213.33\ (\text{cm}^2)$ ***A2***

Note: Answer must be given to 2 decimal places to award ***A2***. Award ***A1A0*** for a correct answer given to an incorrect accuracy of at least 3 significant figures, e.g. $213\ (\text{cm}^2)$.

[4 marks]

(c) attempt to substitute their parts (a) and (b)(ii) into percentage error formula ***(M1)***

$$\left|\frac{213.333... - 200}{213.333...}\right| \times 100$$

$$= 6.25(\%)\ \ (6.24999...(\%))$$ ***A1***

Note: Award ***(M1)A0*** for a final answer of $-6.25(\%)$ or 0.0625.

[2 marks]

[Total: 8 marks]

10. (a) (i) **METHOD 1**

attempt to find change in height of the ball using gradient ***(M1)***

$\frac{a}{0.43} = (-)0.045$

$a = (-)0.045 \times 0.43$

$a = (-)0.0194\,(\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

METHOD 2

attempt to find height at back of home plate ***(M1)***

horizontal distance to the front of the home plate $= 16.6666\ldots\ (\text{m})$

height at the back of the home plate $= -0.045(16.6666\ldots + 0.43) + 2$

$(= 1.23065\ (\text{m}))$

Note: The ***M1*** can be awarded for $16.6666\ldots + 0.43$ seen at some point.

$(a = 1.25 - 1.23065\ldots)$

$(a =)\ (-)0.0194\ (\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

(ii) $1.25 - 0.01935 = 1.23065$ (may be seen in part (a)(i)) ***A1***

$0.53 < 1.23065 < 1.24$ ***R1***

therefore a strike ***AG***

Note: Do not award ***A0R1***.

[4 marks]

continued…

Question 10 continued

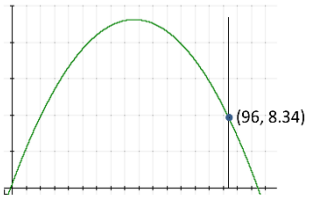
(b) **METHOD 1**

indication of $d = 96$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$(h(96) =) -0.01(96)^2 + 1.04(96) + 0.66$

OR



THEN

$(h(96) =)\ 8.34\ (\text{m})$ ***A1***

$8.34 > 5$ so the ball will go over the wall. ***A1***

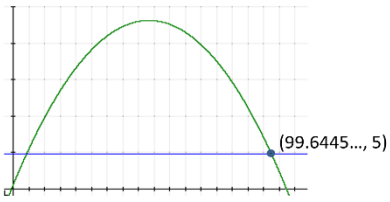
METHOD 2

indication of $h = 5$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$5 = -0.01d^2 + 1.04d + 0.66$

OR



THEN

$d = 99.6\ (\text{m})\ (99.6445\ldots\ (\text{m}))\ (d = 4.35548\ldots\ (\text{m})$ may also be seen) ***A1***

$96 < 99.6445\ldots$ so the ball will go over the wall. ***A1***

[3 marks]

[Total: 7 marks]

11. (a) 14.55 (cm) to 14.65 (cm) ***A1A1***

Note: Award ***A1*** for each value. Accept $14.55 \leq AC < 14.65$.

[2 marks]

(b) attempt to use Pythagorean theorem **OR** trig ratio to find slant height ***(M1)***

a correct expression for either the **upper** or **lower** bound ***(A1)***

$\sqrt{14.55^2 - 10^2}$ **OR** $\sqrt{14.65^2 - 10^2}$ **OR**

$\sin(46.5844...^\circ) = \frac{AH}{14.55}$ **OR** $\sin(46.9533...^\circ) = \frac{AH}{14.65}$

(lower bound =) 10.6 (cm) $(10.5689\ldots)$ **AND**

(upper bound =) 10.7 (cm) $(10.7061\ldots)$ ***A1***

[3 marks]

continued…

Question 11 continued

(c) **METHOD 1**

attempt to find the maximum angle measure of the post using trigonometry ***(M1)***

e.g. $\cos\theta = \frac{10}{10.7061...}$ **OR** $\frac{\sin\theta}{3.82393\ldots} = \frac{\sin(90^\circ)}{10.7061\ldots}$

Note: Accept an inequality.

$(\theta =)\ 20.9\ (^\circ)\quad (20.9265\ldots(^\circ))$ ***A1***

and hence the post is safe ***AG***

Note: Use of radians gives an answer of $0.365\ \ (0.365237\ldots)$; award at most ***(M1)A0*** since this value cannot be directly compared to 22°.

Award at most ***(M1)A0*** for an angle calculated using their lower bound from part (b).

METHOD 2

attempt to find the longest slant height for angle to be a maximum of 22° ***(M1)***

e.g. $\cos(22^\circ) = \frac{10}{x}$

$(x = 10.7853\ldots)$

$10.7061\ldots < 10.7853\ldots$ ***A1***

and hence the post is safe ***AG***

Note: A comparison to their upper bound from part (b) is required for ***A1*** to be awarded. Use of radians gives an unreasonable answer of $-10.0003...$; award at most ***(M1)A0***.

[2 marks]

Total [7 marks]

12. (a) attempt to find the difference between 75.7 and 67.3 ***(M1)***

$\frac{75.7-67.3}{2}$

$4.2\ (\text{km h}^{-1})$ ***A1***

[2 marks]

(b) recognition of normal distribution that includes 72 ***(M1)***

e.g., sketch of normal distribution curve with 72 labelled to the right of the mean **OR**
Normal CDF calculation using 72

$0.132\quad (0.131559\ldots, 13.2\%, 13.1559\ldots\%)$ ***A1***

[2 marks]

(c) **METHOD 1 (Comparing areas above and below the mean)**

$\text{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF$(67.3, 74, 67.3, 4.2)$ **OR** sketch of normal distribution with 67.3 and 74 labelled and shaded between ***(M1)***

area of region between mean and q is at least $0.445\ \ (0.444670\ldots)$ ***A1***

Hence no more than $0.375\ \ (0.375329\ldots)$ between mean and p ***R1***

The region between p and q is not symmetrical ***AG***

METHOD 2 (Comparing areas in the tails)

attempt to calculate probability that speed $< p$ and speed $> q$ with $q = 74$ ***(M1)***

$\text{P}(\text{speed} < 74) = 0.944670\ldots$

$\text{P}(\text{speed} < p) = (0.944670\ldots - 0.82 =)\ \ 0.124670\ldots$

$\text{P}(\text{speed} > q) = (1 - 0.944670\ldots =)\ \ 0.0553295\ldots$ ***A1***

if $q \geq 74$, then $\text{P}(\text{speed} > q) \leq 0.0553295$ and $\text{P}(\text{speed} < p) \geq 0.124670$ so

$\text{P}(\text{speed} > q)$ will never equal $\text{P}(\text{speed} < p)$ ***R1***

the region between p and q is not symmetrical ***AG***

continued…

Question 12 continued

METHOD 3 (Assumption of symmetry comparing speeds)

attempt to calculate area below q assuming distribution is symmetrical ***(M1)***

e.g. $\mathrm{P}(\text{speed} < q) = 0.82 + ½ \times 0.18 \quad (0.91)$

EITHER

$(q =)\ 72.9 \quad (72.9311\ldots)$ ***A1***

$72.9 < 74$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$\mathrm{P}(\text{speed} < 74) = 0.945 \quad (0.944670\ldots)$ ***A1***

$0.945 > 0.91$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

METHOD 4 (Assumption of symmetry comparing areas)

attempt to calculate symmetrical area with 74 as a boundary ***(M1)***

$\mathrm{P}(60.6 < \text{speed} < 74)$ **OR** Normal CDF(60.6, 74, 67.3, 4.2) **OR**

$\mathrm{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF(67.3, 74, 67.3, 4.2)

EITHER

$0.889 \ (0.889340\ldots)$ ***A1***

$0.889 > 0.82$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

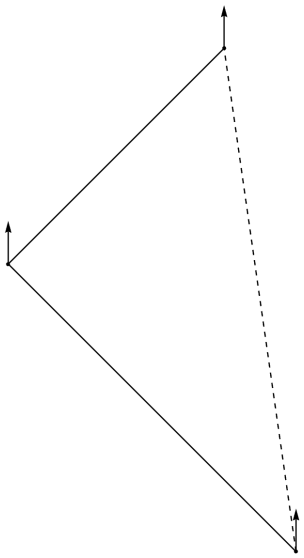
$0.445 \ (0.444670\ldots)$ ***A1***

$0.445 > 0.82 \div 2$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

[3 marks]
[Total: 7 marks]

13. diagram showing (approximately) correct directions (and order) for the 315° and 045° ***(A1)***



Note: Values do not need to be seen on the diagram to award the ***A1***.

recognizing right angle triangle ***(M1)***

correct expression to find second angle in triangle ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

correct expression to find bearing ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)+135^\circ$ **OR** $360^\circ-\left(\arctan\left(\frac{8}{6}\right)+135^\circ\right)$

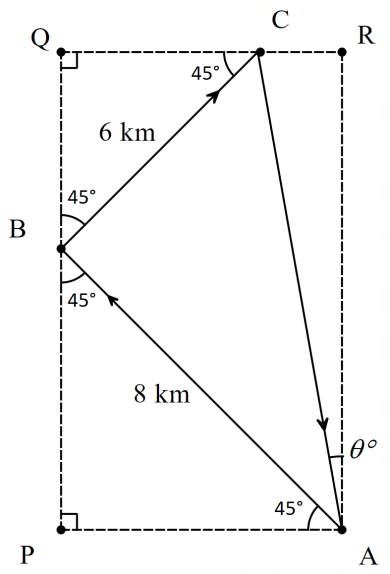
$=172^\circ$ $(171.869...^\circ)$ ***A1***

continued…

Question 13 continued

METHOD 2

diagram showing (approximately) correct directions (and order) for the $315°$ and $045°$ (these may be shown in reverse as the return journey) ***(A1)***



finding the lengths marked $\mathrm{AP}, \mathrm{BP}, \mathrm{CQ}$ and BQ in the diagram ***(M1)***

$$\mathrm{AP} = \mathrm{BP} = 8\frac{\sqrt{2}}{2} = 5.6568...$$

$$\mathrm{CQ} = \mathrm{BQ} = 6\frac{\sqrt{2}}{2} = 4.2426...$$

Note: This may be done using a vector approach.

using $\tan\theta° = \dfrac{\mathrm{AP} - \mathrm{CQ}}{\mathrm{PB} + \mathrm{BQ}}$ or equivalent to find the direction of AC ***(A1)***

correct expression to find bearing ***(A1)***

$$180° - \arctan\left(\frac{8\frac{\sqrt{2}}{2} + 6\frac{\sqrt{2}}{2}}{8\frac{\sqrt{2}}{2} - 6\frac{\sqrt{2}}{2}}\right)$$

$= 172° \quad (171.869...°)$ ***A1***

[Total: 5 marks]