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Mathematics: applications and interpretation

Standard level

Paper 2

9 May 2023

Zone A afternoon | Zone B morning | Zone C afternoon

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

The mean annual temperatures for Earth, recorded at fifty-year intervals, are shown in the table.

Year (x)	1708	1758	1808	1858	1908	1958	2008
Temperature °C (y)	8.73	9.22	9.10	9.12	9.13	9.45	9.76

Tami creates a linear model for this data by finding the equation of the straight line passing through the points with coordinates (1708, 8.73) and (1958, 9.45).

- (a) Calculate the gradient of the straight line that passes through these two points. [2]
- (b) (i) Interpret the meaning of the gradient in the context of the question.
- (ii) State appropriate units for the gradient. [2]
- (c) Find the equation of this line giving your answer in the form $y = mx + c$. [2]
- (d) Use Tami's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses linear regression to obtain a model for the data.

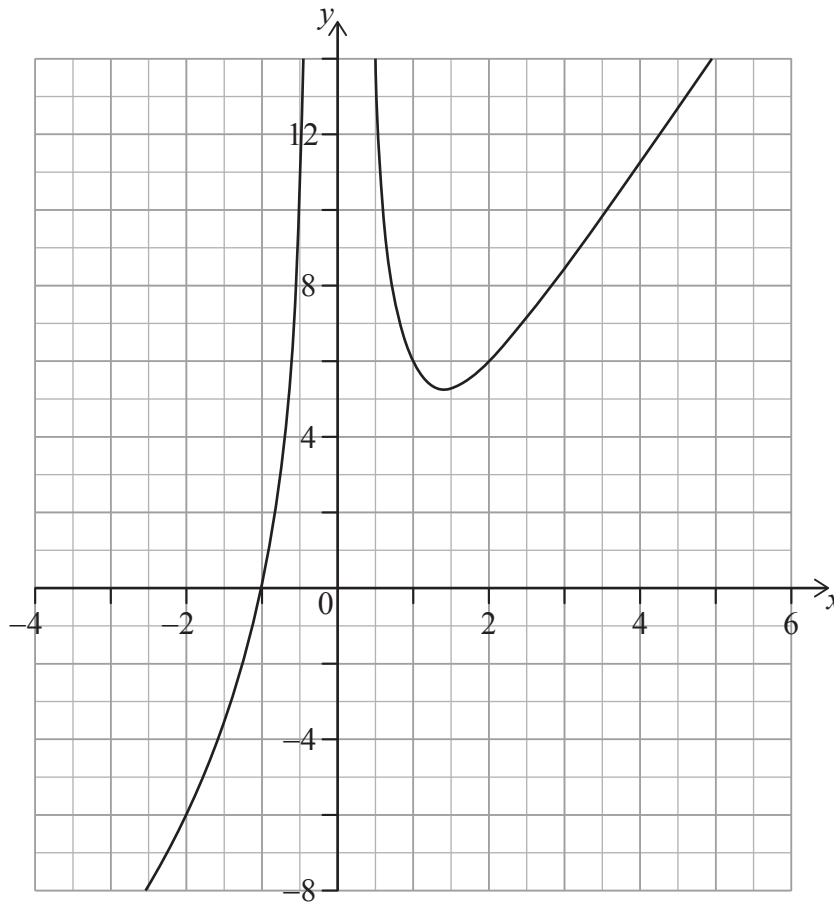
- (e) (i) Find the equation of the regression line y on x .
- (ii) Find the value of r , the Pearson's product-moment correlation coefficient. [3]
- (f) Use Thandizo's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses his regression line to predict the year when the mean annual temperature will first exceed 15 °C.

- (g) State two reasons why Thandizo's prediction may not be valid. [2]

2. [Maximum mark: 16]

Consider the function $f(x) = 3x - 1 + 4x^{-2}$. Part of the graph of $y = f(x)$ is shown below.



The function is defined for all values of x except for $x = a$.

(a) Write down the value of a . [1]

(b) Use your graphic display calculator to find the coordinates of the local minimum. [2]

The equation $f(x) = w$, where $w \in \mathbb{R}$, has three solutions.

(c) Identify one possible value for w . [1]

The line $y = mx - \frac{1}{4}$ is tangent to $f(x)$ when $x = -4$.

(d) Write down whether the value of m is positive or negative. Justify your answer. [2]

(This question continues on the following page)

(Question 2 continued)

A second function is given by $g(x) = kp^x - 9$, where $p > 0$. The graph of $y = g(x)$ intersects the y -axis at point $A(0, -5)$ and passes through point $B(3, 4.5)$.

- (e) Find the value of
 - (i) k ;
 - (ii) p . [4]
- (f) Write down the equation of the horizontal asymptote of $y = g(x)$. [2]
- (g) Find the solution of $f(x) = g(x)$ when $x > 0$. [2]

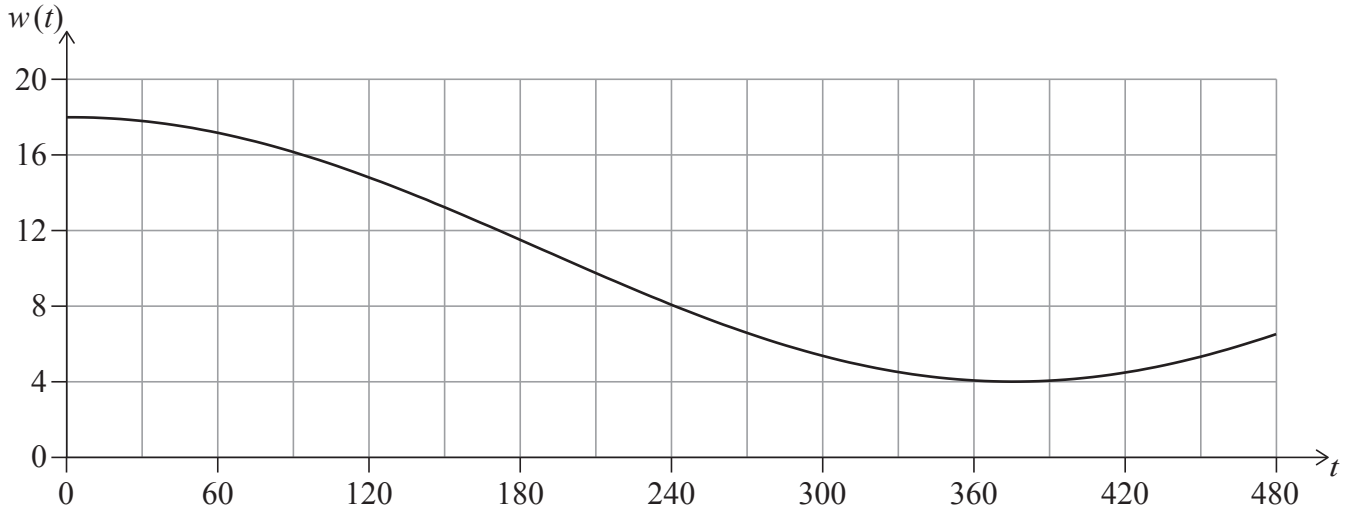
Consider a third function, h , where $h(x) = f(x) + g(x)$. The point $C(-1, q)$ lies on the graph of $g(x)$.

- (h) State whether C also lies on the graph of $h(x)$. Justify your answer. [2]

3. [Maximum mark: 15]

The depth of water, w metres, in a particular harbour can be modelled by the function $w(t) = a \cos(bt^\circ) + d$ where t is the length of time, in minutes, after 06:00.

On 20 January, the first high tide occurs at 06:00, at which time the depth of water is 18 m. The following low tide occurs at 12:15 when the depth of water is 4 m. This is shown in the diagram.



- (a) Find the value of a . [2]
- (b) Find the value of d . [2]
- (c) Find the period of the function in minutes. [3]
- (d) Find the value of b . [2]

Naomi is sailing to the harbour on the morning of 20 January. Boats can enter or leave the harbour only when the depth of water is at least 6 m.

- (e) Find the latest time before 12:00, to the nearest minute, that Naomi can enter the harbour. [4]
- (f) Find the length of time (in minutes) between 06:00 and 15:00 on 20 January during which Naomi **cannot** enter or leave the harbour. [2]

4. [Maximum mark: 17]

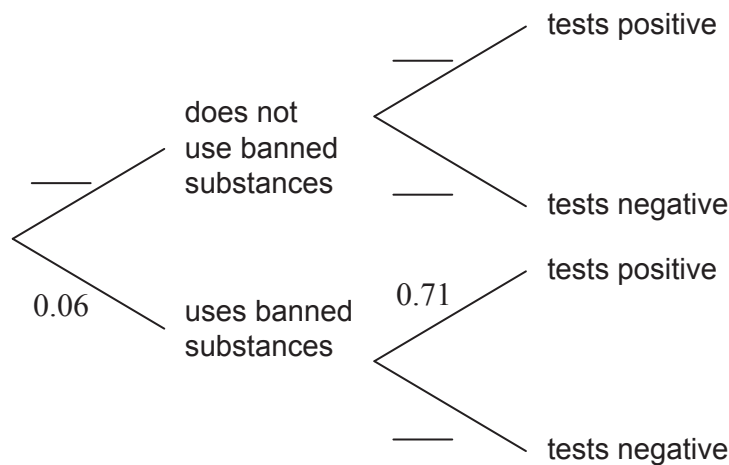
A large international sports tournament tests their athletes for banned substances. They interpret a positive test result as meaning that the athlete uses banned substances. A negative result means that they do not.

The probability that an athlete uses banned substances is estimated to be 0.06.

If an athlete **uses** banned substances, the probability that they will test positive is 0.71.

If an athlete does **not use** banned substances, the probability that they will test negative is 0.98.

- (a) Using the information given, **copy** (into your answer booklet) and complete the following tree diagram. [2]



- (b) (i) Determine the probability that a randomly selected athlete does not use banned substances and tests negative.
- (ii) If two athletes are selected at random, calculate the probability that both athletes do not use banned substances and both test negative. [4]
- (c) (i) Calculate the probability that a randomly selected athlete will receive an **incorrect** test result.
- (ii) A random sample of 1300 athletes at the tournament are selected for testing. Calculate the expected number of athletes in the sample that will receive an incorrect test result. [5]

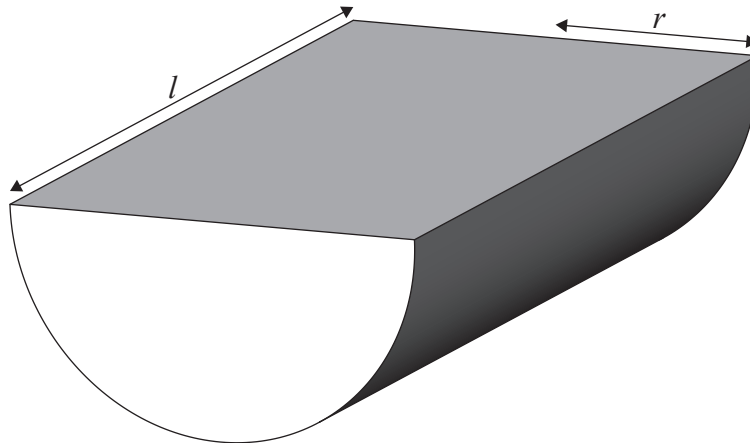
Team X are competing in the tournament. There are 20 athletes in this team. It is known that none of the athletes in Team X use banned substances.

- (d) Calculate the probability that none of the athletes in Team X will test positive. [4]
- (e) Determine the probability that more than 2 athletes in Team X will test positive. [2]

5. [Maximum mark: 17]

A large closed container, in the shape of a half cylinder with a rectangular lid, is to be constructed with a volume of 0.8 m^3 . The container has a length of l metres and a radius of r metres.

diagram not to scale



- (a) Find an exact expression for l in terms of r and π . [2]

The container will be constructed using two different materials. The material for both the curved surface and the rectangular lid of the container costs \$4.40 per square metre. The material for the semicircular ends of the container costs $\$p$ per square metre.

The cost, C , of the materials to construct the container can be written in terms of r and p (where $p > 0$ and $r > 0$).

- (b) Show that $C = 7.04r^{-1} + \frac{14.08}{\pi}r^{-1} + p\pi r^2$. [4]

- (c) Find $\frac{dC}{dr}$. [3]

The cost of materials to construct the container is minimized when the radius of the container, r , is 0.7 m.

- (d) Find the value of p . [3]

In total, 350 containers will be constructed at this minimum cost.

- (e) Calculate the cost of materials, to the nearest dollar, to construct all 350 containers. [3]

(This question continues on the following page)

(Question 5 continued)

The materials for constructing the containers can be purchased at a discount according to the information in the table.

Cost of materials (\$ C) before discount	Discount applied to entire order
$1000 \leq C < 2500$	1%
$2500 \leq C < 5000$	4%
$5000 \leq C < 10\,000$	8%
$C \geq 10\,000$	10%

- (f) Determine the cost of materials for 350 containers after the discount is applied. [2]
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References: